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Analysis of hammer movement based on a parametrically excited pendulum model

Ken Ohta[a,*], Koji Umegaki[b], Koji Murofushi[c], ZhiWei Luo[d]

[a] *Japan Institute of Sports Science, 3-15-1 Nishigaoka, Kita, Tokyo 115-0056, Japan*
[b] *Maizuru National College of Technology, 234 Shiroya Maizuru, Kyoto 625-8511, Japan*
[c] *Chukyo University, 101 Tokodachi, Kaizu, Toyota, Aichi, 470-0393, Japan*
[d] *Kobe University, 1-1 Rokkodaicho, Nada, Kobe 657-8501, Japan*

Abstract

Motions of the hammer were analyzed to understand the mechanism of acceleration with a hula-hoop model using an energy pumping mechanism. The condition that makes the time derivative of the energy positive is derived as energy pumping for hammer. The condition is expressed in terms of the tugging force times velocity to pump the hammer energy. In this study, motions of the hammer were analyzed and numerical experiments were performed to examine the validity of the theory.

Keywords: Hammer throw, Hula-hoop, Energy pumping, Hetero-parametric excitation

1. Introduction

The motions of a hammer are commonly regarded as a circular movement which a hammer undergoes around a turning human body. The distance of a throw depends on the speed of the hammer and the angle of its trajectory at the instant of release. The speed of the hammer increases gradually during preliminary winds and turns. The hammer throw is a sequence 1 of turns that involves moving the hammer while turning on either one leg (single-support phase) and turning on both legs (double-support phase)(Fig. 1). During each turn the hammer rises up toward the high point of the trajectory and passes through the low point. There are many studies which describe such hammer and thrower movement with experimental analysis [1, 2, 3, 4]. However, theoretical analysis which describe the mechanisms of acceleration are few. Our interest in this study was to understand the mechanism of acceleration of the hammer in terms of energy pumping. The motions of hammer were analyzed and numerical experiments were performed to examine our theory.

*Corresponding author, Tel: +81-3-5963-0221, fax: +81-3-5963-0222
Email address: ohta@jiss.naash.go.jp (Ken Ohta)

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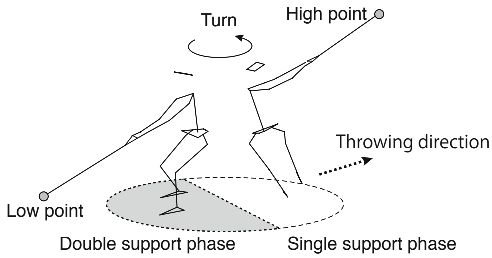

Figure 1: Double-support and single-support phases

2. Methods

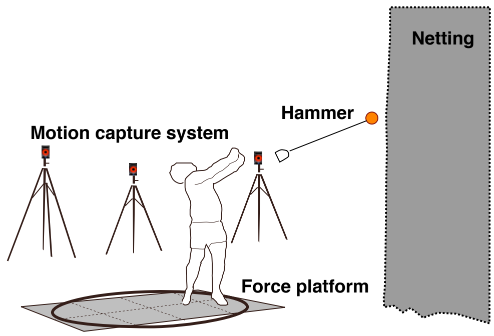

Figure 2: Experimental setup

One elite highly skilled thrower performed seven trials. Six markers (14mm diameter) were put on the hammer and the 3D positions of the hammer were obtained by a motion analysis system (Vicon, Oxford, UK) composed of 15 cameras and sampled at 250 Hz. Eight force platforms (EFP-S-10kNSA5, Kyowa electronic instruments, JPN) were used to capture ground reaction force data at a sampling rate of 1000 Hz. Smoothing Spline functions were used to smooth the time-dependent position vector data of each hammer marker. Fig. 2 shows the experimental setup. The athlete was asked to throw a hammer from a throwing circle in which the force platforms were embedded toward netting as a buffer.

3. Results

Angular velocity, angular acceleration, velocity and radius of curvature of the hammer were calculated (Fig. 3). The radius of curvature is given by

$$r(t) = \frac{\|\dot{\boldsymbol{X}}(t)\|^3}{\|\dot{\boldsymbol{X}}(t) \times \ddot{\boldsymbol{X}}(t)\|}, \tag{1}$$

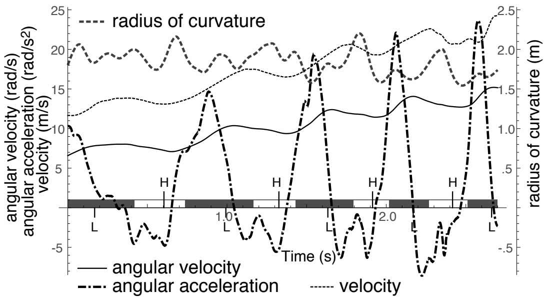

Figure 3: Angular velocity, angular acceleration, linear velocity and radius of curvature of the hammer during turns.

where $\boldsymbol{X}(t)$ is the position vector of the hammer head. The shaded and white bars on the abscissa axis of each graph indicate double- and single-support phases, respectively. The vertical lines on the same axis represent the low points during double-support phase and the high points during single-support phases. Although the velocity and the angular velocity increased gradually, they both increased not monotonically but in an oscillating way over the course of the turns. The local maximums of the angular velocity did not coincide with one of the velocity and phase delays were seen between the angular velocity profile and the linear velocity one.

4. Hula-hoop model

4.1. System dynamics

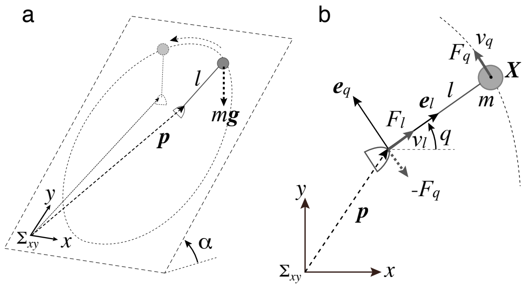

Figure 4: Hammer model

To study the hammer movement and energy pumping, a hula-hoop model is employed (Fig. 4). The pendulum moves on an inclined plane at a fixed angle α and has mass m and an inextensible weightless wire of length l. The hammer moves in the Cartesian coordinates Σ_{xy} on the plane. The motion degrees of freedom of the pendulum are represented by the position vector of the handle $\boldsymbol{p}(t) = [x_0(t),\ y_0(t)]^T$ and the counterclockwise angle of deviation of the pendulum $q(t)$ from the x axis. The hammer handle moves with circular movement similar to a hula-hoop motion during the turns.

The position vector of the hammer is

$$\boldsymbol{X}(t) = \boldsymbol{p}(t) + l\,\boldsymbol{e}_l(t) \tag{2}$$

and it's velocity vector is

$$\dot{\boldsymbol{X}}(t) = \dot{\boldsymbol{p}}(t) + l\,\dot{q}(t)\boldsymbol{e}_q(t), \tag{3}$$

where $\boldsymbol{e}_l(t) = [\cos q(t),\ \sin q(t)]^T$ is a unit vector in the normal direction and $\boldsymbol{e}_q(t) = [-\sin q(t),\ \cos q(t)]^T$ is the tangential unit vector. The kinetic energy T and potential energy U

$$T = \frac{1}{2}m\dot{\boldsymbol{X}}^T\dot{\boldsymbol{X}} = \frac{1}{2}m(\dot{x}_0^2 + \dot{y}_0^2 + l^2\dot{q}^2 + 2l\dot{q}\boldsymbol{e}_q^T\dot{\boldsymbol{p}}), \tag{4}$$

$$U = mg_\alpha(y_0 + l\sin q), \tag{5}$$

are obtained to employ system dynamics by applying Lagrange's equation and the system dynamics

$$ml\ddot{q} + m\boldsymbol{e}_q^T(\ddot{\boldsymbol{p}} - \boldsymbol{G}_\alpha) = 0, \tag{6}$$

$$m\boldsymbol{e}_l^T(\ddot{\boldsymbol{p}} - \boldsymbol{G}_\alpha) - ml\dot{q}^2 = F_l, \tag{7}$$

can be derived, where $g_\alpha \equiv g\sin\alpha$, $\boldsymbol{G}_\alpha = [0, -g_\alpha]^T$ and F_l is the wire tensile force. The tangential dynamics (6) shows that $-\boldsymbol{e}_q^T\ddot{\boldsymbol{p}}$, which is in the opposite direction of the hammer head, increases the angular acceleration of the hammer.

4.2. Energy puming in the normal dynamics

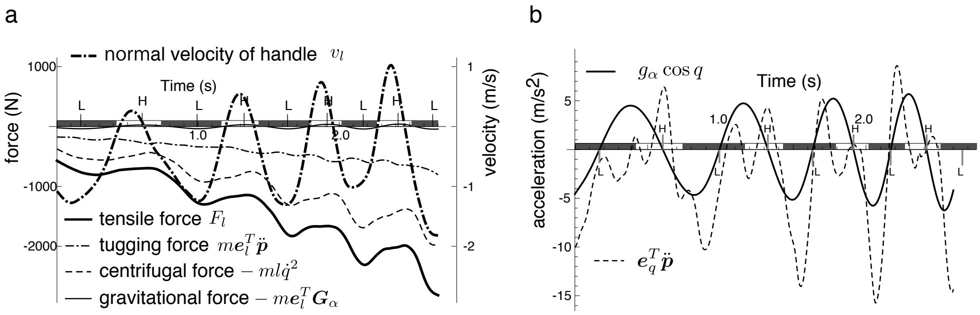

Figure 5: a) Wire tensile force and normal velocity of the handle. b) Comparison of tangential components of gravity $g_\alpha \cos q$ and acceleration $\boldsymbol{e}_q^T\ddot{\boldsymbol{p}}$

The mechanical energy E

$$E = T + U \tag{8}$$

is derived to understand the principle of acceleration. The time derivative of the energy

$$\dot{E} = F_l v_l \tag{9}$$

is given by considering Eqs. (4), (5), (6), (7), where $v_l \equiv \boldsymbol{e}_l^T\dot{\boldsymbol{p}}$ is the normal velocity of the handle.

The power $\dot{E}$ has only the normal component $F_l v_l$. The instantaneous power $\dot{E}$ must be positive to increase mechanical energy of the hammer and can be positive by giving negative velocity ($v_l < 0$) under the condition of the negative F_l during turns. Fig. 5a shows that most of the signs of the velocity v_l are negative throughout the course of turns. This means the energy increases over the duration of turns because F_l is constantly negative. The mechanical energy of the hammer in each turn was increased with a fluctuation, since the oscillating tensile force tends to increase in the double-support phase and decrease in the single-support phase. Maximizing the restored energy in each turn is one of the strategies to maximize the mechanical energy over the duration of turns. Both the tugging force which causes negative v_l at the low point and the decreasing wire tensile force which causes positive v_l near the high-point are optimal ways to yield a maximized normal component of restored energy

$$\Delta E = \int \dot{E} dt = \int F_l v_l \tag{10}$$

in each turn, because the tensile force F_l reaches a local maximum at the low point and reaches a local minimum near the high point. This theoretical approach for restoring kinetic energy using parametric excitation, which is a principle to increase energy by controlling velocity v_l [5, 6], can be applied to the hammer throw and this energy pumping method can be seen in Fig. 5a.

4.3. Parametric excitation in the tangential dynamics

The time derivative of the energy $\dot{E}$ depends on the normal dynamics, however, the tensile force F_l strongly depends on the centrifugal force which is determined by the tangential dynamics (6).

To analyze the tangential dynamic behavior,

$$\ddot{q} + \frac{1}{l}\boldsymbol{e}_q^T(\ddot{\boldsymbol{p}} - \boldsymbol{G}_\alpha) = 0 \tag{11}$$

$$\ddot{q} + \frac{1}{l}(\boldsymbol{e}_q^T\ddot{\boldsymbol{p}} + g_\alpha \cos q) = 0 \tag{12}$$

were derived from the dynamics (6). Assuming periodic tangential movement of the handle, Eq. (12) can be transformed into a Mathieu equation with non-linear restoring force which describes parametric excitation [7].

Fig. 5b shows the tangential handle acceleration $\boldsymbol{e}_q^T\ddot{\boldsymbol{p}}$ and tangential gravitational acceleration $g_\alpha \cos q$ during the course of turns. The phase of the handle acceleration $\boldsymbol{e}_q^T\ddot{\boldsymbol{p}}$ coincides roughly with one of gravitational accelerations $g_\alpha \cos q$ and this indicates that the hammer was pumped up by parametrical excitation.

4.4. Simulation

Numerical simulation was performed to examine the effect of the parametric excitation using Eqs. (6),(7). Fig. 6 shows the simulation results under the conditions,

$$F_q \equiv \boldsymbol{e}_q^T\ddot{\boldsymbol{p}} = k_\alpha \xi(\cos q), \tag{13}$$

$$F_l = -R(1-\beta t)\dot{q}^2 - k_\gamma \sin q, \tag{14}$$

$$\xi(x) \equiv \begin{cases} x & : x \geq 0 \\ 0.5x & : x < 0 \end{cases} \tag{15}$$

and $k_\alpha = 18, R = 1.7$ m, $\beta = 0.02, k_\gamma = 0.5$. In this simulation, the control input was given in phase with the tangential component of gravity to excite the circular movement by means of parametric excitation. Fig. 6 shows the results of the numerical simulation which is similar to the experimental results shown in Fig. 3.

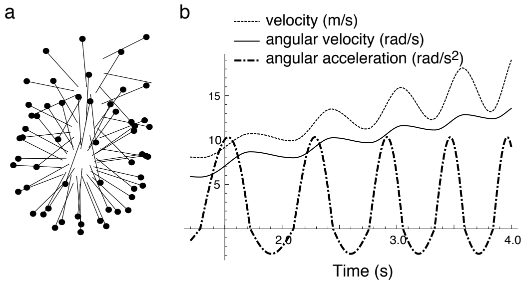

Figure 6: Simulation results. a) Stick pictures of hammer. b) Kinematic data ($\|\dot{X}\|, \dot{q}, \ddot{q}$).

5. Conclusion

The motions of hammer were analyzed to understand the mechanism of acceleration with a hula-hoop model using an energy pumping mechanism. The condition that makes the time derivative of the energy positive is derived as energy pumping for the hammer and the condition is expressed in terms of the tugging force times velocity to pump hammer energy.

As far as normal direction, tugging near the low point gives the optimal way to yield maximized restored energy in each turn, because the tensile force F_l reaches a local maximum near the low point. This is an approach for restoring kinetic energy using parametric excitation which is a principle to increase energy. Giving a tangential acceleration in phase with gravity using another type of parametric excitation yields a larger F_l near the low point and this maximizes this energy pumping effect. Optimal coupling of the hetero-parametric excitations and application as biofeedback system for the hammer training [8] are our further researches.

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